## 1 Particle Number

The number of particles tracked, the grid size and the averaging time,  $T_{ave}$ , determines the threshold concentration which can be predicted by Hysplit. The smallest concentration which the model can compute for  $T_{ave} = 0$  is  $\frac{m_p}{v}$  where  $m_p$  is the mass of one particle and v is the volume of one grid cell.

Suppose we have a fairly large eruption with an MER= $1 \times 10^7$  kg/s and we assume the mass fraction of fine ash is a tenth of that. In our hysplit simulation we assume that one unit mass of material is released per hour. Converting the MER to grams/hour and taking a tenth of that number we find that the unit mass is equal to  $3.6 \times 10^{12}$  grams. Supposing we run with 40,000 particles released per hour and 4 different particle sizes. 10,000 of each particle size is released every hour.

Typical vertical resolution is about 3-6 km (10-000 - 20,000 ft). Typical horizontal resolution is between 0.05 and 0.2 degrees latitude/longitude (5-22 km) Volume of one grid cell could range from 90 to 3000 km<sup>3</sup>.

- $20\mu \text{m}$  particles have 67% of the mass  $(2.4 \times 10^{12} \text{g})$  and  $m_{p20} = 2.4 \times 10^8 \text{g}$
- $6\mu$ m particles have 25.4% of the mass  $(9.1 \times 10^{11} \text{g})$  and  $m_{p06} = 9.1 \times 10^{7} \text{g}$
- $2\mu$ m particles have 6.8% of the mass  $(2.5 \times 10^{11} \text{g})$  and  $m_{p02} = 2.5 \times 10^7 \text{g}$

 $2.6 \,\mu\text{m}$  particles have 0.5% of the mass  $(2.6 \times 10^{-6} \text{ g})$  and  $m_{p02} = 2.5 \times 10^{-6} \text{ g}$   $0.6 \,\mu\text{m}$  particles have 0.8% of the mass  $(2.9 \times 10^{-10} \text{ g})$  and  $m_{p006} = 2.9 \times 10^{6} \text{ g}$ For a 90 km<sup>3</sup> cell  $\frac{m_{p20}}{v} = 2.6 \frac{\text{mg}}{\text{m}^3} \frac{m_{p06}}{v} = 1 \frac{\text{mg}}{\text{m}^3} \frac{m_{p02}}{v} = 0.27 \frac{\text{mg}}{\text{m}^3} \frac{m_{p006}}{v} = 0.03 \frac{\text{mg}}{\text{m}^3}$ If the threshold concentration of interest is  $2 \frac{\text{mg}}{\text{m}^3}$  this combination of grid size and particle number would be problematic. Regions which show concentrations below  $2.6 \frac{\text{mg}}{\text{m}^3}$  would be limited to regions which do not contain the largest sized particles.

It is desireable that the particle number and grid size be chosen so that  $\frac{n_{min}m_p}{v} = c_t$ , where  $n_{min}$  is some minimum number of the largest size of particles and  $c_t$  is the smallest concentration of interest. In the case of volcanic ash concentrations above  $2\frac{mg}{m^3}$  are currently estimated to be the amount dangerous to aircraft. Because of uncertainties in determining source term parameters, models should be able to forecast concentrations one to two orders of magnitude below the amount considered dangerous. Therefore we will take  $c_t = 0.02 \frac{\text{mg}}{\text{m}^3}$ . The minimum number,  $n_{min}$ , of particles in a concentration grid needs to be determined. Since particles have a random component to their motion, the number of particles in any particular grid box may be expected to change somewhat from one simulation run to another even when all inputs remain the same. Suppose the number of particles in a grid box may be expected to fluctuate by  $\pm n_f$ . It would be

desireable for  $\frac{n_f}{n_{min}}$  to be small. Averaging over time  $T_{ave}$  will decrease the number of particles needed. Supposing the concentration does not change significantly over the averaging time  $T_{ave}$ . The average residence time,  $t_r$ , of a particle in a grid box would be the width of the grid cell divided by the mean velocity of the particle. If you averaged over time  $T_{ave}$  then the number of particles you would need could be decreased by about  $T_{ave}/t_r$ . Care should be taken when relying on time averaging since different regions of the ash cloud will probably have different residence times.

If concentration is changing significantly over time  $T_{ave}$  then averaging could be seen as a type of ensemble forecast that accounts for uncertainties in the timing of the event. However care would need to be taken in interpreting the quantitative value of the concentration.

If the concentration gradient is very steep then adding more particles may not increase the area with concentrations above  $c_t$  by very much. The concentration gradient will be dependent on the dispersion. In this sense the puff model may have some advantages since the number of puffs will increase as the cloud disperses. Therefore computational resources will not be wasted in tracking many particles which are all very close together at the beginning of the run. On the other hand it is much easier to predict the computational resources needed to track a known number of particles. Until the model is run it is not known how many puffs will need to be tracked and indeed the number of puffs needed may vary depending on input parameters.

Using some type of kernel to represent the position of each particle could also be an effective way to reduce the number of particles which need to be tracked since it would tend to smooth out random fluctuations in the number of particles per grid box.