

# Source Attribution Methods

HYSPLIT Workshop 2022, Jun. 16, 2022

Version 5.2.1 (revision 1694)

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# 13.1 Emissions from a Known Location

Concentration (M) is the product of the atmospheric dilution (D) from the source to the receptor and the emission rate (Q):

$$\text{Counterpart of the measurement (M): } C [\text{g/m}^3] = D [\text{hr/m}^3] Q [\text{g/hr}]$$

Because the model computes the dilution factor, we can simply re-arrange the equation and adjusting for units :

$$Q [\text{g/hr}] = M [\text{pg/m}^3] / ( D [\text{hr/m}^3] * 1\text{E}+12 [\text{pg/g}])$$

3-hr release from 17Z Sep. 25, 1983

$$Q = 67 \text{ Kg/hr} * (2303/759) = 220 \text{ Kg/hr}$$

```
Contents of statA.txt ...  
  
C:/Users/Ti results file: statA.txt  
Model variation: Tracer number: 0 Station select: All  
-----  
  
48 Unaveraged data points for processing  
0.00 Percentile input for zero measured  
0.00 Zero measured concentration value  
  
0.91 Correlation coefficient (P=99%)  
0.25 Regression Slope  
14.81 T-value (|Slope|/Standard Error)  
2303.92 Average measured concentration  
759.84 Average calculated concentration  
0.33 Ratio of calculated/measured  
12.14 Normalized mean square error  
4609.71 Root mean square error  
48 Number of pairs analyzed
```

21-hr release from 15Z Sep. 25, 1983

$$Q = 1 \text{ g/hr} * (2303/0.03)/1000 = 77 \text{ Kg/hr}$$

```
Contents of statA.txt ...  
  
C:/Users/Ti results file: statA.txt  
Model variation: Tracer number: 0 Station select: All  
-----  
  
48 Unaveraged data points for processing  
0.00 Percentile input for zero measured  
0.00 Zero measured concentration value  
  
0.82 Correlation coefficient (P=99%)  
0.00 Regression Slope  
9.63 T-value (|Slope|/Standard Error)  
2303.92 Average measured concentration  
0.03 Average calculated concentration  
0.00 Ratio of calculated/measured  
507828.03 Normalized mean square error  
6168.21 Root mean square error  
48 Number of pairs analyzed
```

# 13.2 Backward versus Forward Dispersion

Forward run from S001 (39.90,-82.22)  
 1-hr release from **83 09 25 17**  
 Dilution factor ( $\times 1E15$ ) at S316

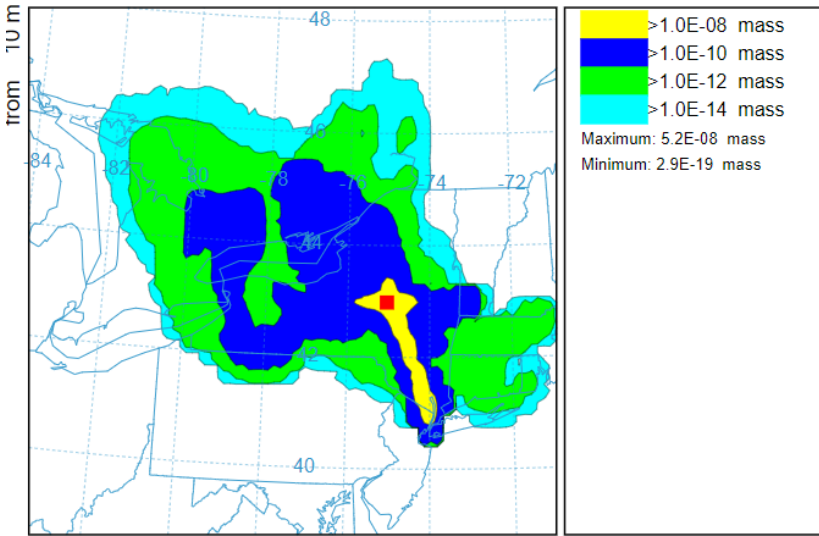
JDAY	YR	MO	DA1	HR1	MN1	DA2	HR2	MN2	S316
268.708	83	9	25	17	0	25	18	0	0.
...									
268.958	83	9	25	23	0	26	0	0	0.
269.000	83	9	26	0	0	26	1	0	0.259
269.042	83	9	26	1	0	26	2	0	0.431
269.083	83	9	26	2	0	26	3	0	1.378
269.125	83	9	26	3	0	26	4	0	4.564
269.167	83	9	26	4	0	26	5	0	12.057
269.208	83	9	26	5	0	26	6	0	19.288
269.250	83	9	26	6	0	26	7	0	30.080
<b>269.292</b>	<b>83</b>	<b>9</b>	<b>26</b>	<b>7</b>	<b>0</b>	<b>26</b>	<b>8</b>	<b>0</b>	<b>30.772</b>
269.333	83	9	26	8	0	26	9	0	22.274
269.375	83	9	26	9	0	26	10	0	17.334
269.417	83	9	26	10	0	26	11	0	9.872
269.458	83	9	26	11	0	26	12	0	4.247
269.500	83	9	26	12	0	26	13	0	1.378
269.542	83	9	26	13	0	26	14	0	0.115

Backward run from S316 (41.30, -84.22)  
 1-hr release from **83 09 26 08**  
 Dilution factor ( $\times 1E15$ ) at S001

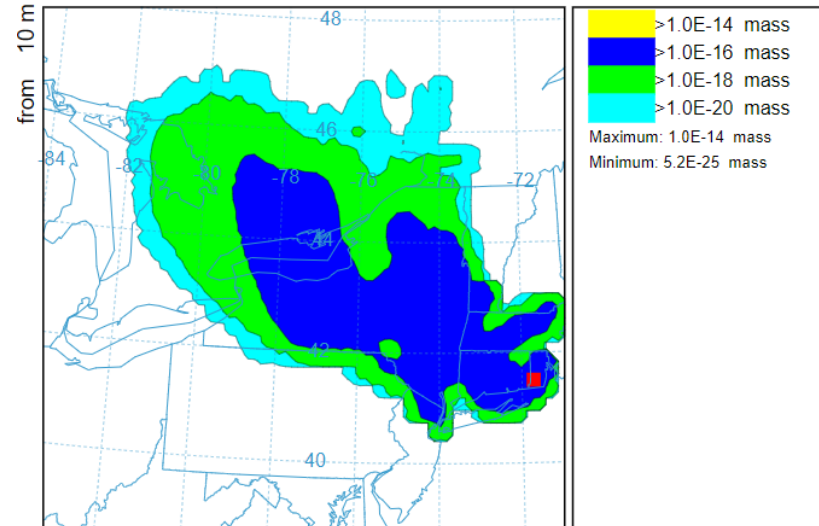
JDAY	YR	MO	DA1	HR1	MN1	DA2	HR2	MN2	S001
269.333	83	9	26	8	0	26	7	0	0.
269.292	83	9	26	7	0	26	6	0	0.
269.250	83	9	26	6	0	26	5	0	0.
269.208	83	9	26	5	0	26	4	0	0.
269.167	83	9	26	4	0	26	3	0	0.
269.125	83	9	26	3	0	26	2	0	0.
269.083	83	9	26	2	0	26	1	0	0.
269.042	83	9	26	1	0	26	0	0	0.
269.000	83	9	26	0	0	25	23	0	0.
268.958	83	9	25	23	0	25	22	0	0.
268.917	83	9	25	22	0	25	21	0	5.913
268.875	83	9	25	21	0	25	20	0	24.104
268.833	83	9	25	20	0	25	19	0	21.284
268.792	83	9	25	19	0	25	18	0	18.238
<b>268.750</b>	<b>83</b>	<b>9</b>	<b>25</b>	<b>18</b>	<b>0</b>	<b>25</b>	<b>17</b>	<b>0</b>	<b>9.679</b>

# 13.3 Emissions from an Unknown Location

Weighted Source Sensitivity Function  
 Receptor ( mass ) averaged between 0 m and 100 m  
 Integrated from 1200 03 Sep to 0000 01 Sep 83 (UTC) [backward]  
 MEAN Calculation started at 2100 01 Sep 83 (UTC)



Inverse of the Averaged Emissions  
 Receptor ( mass ) averaged between 0 m and 100 m  
 Integrated from 1200 03 Sep to 0000 01 Sep 83 (UTC) [backward]  
 MEAN Calculation started at 2100 01 Sep 83 (UTC)



CDC1 METEOROLOGICAL DATA

Hypothetical measurements (29)  
 were generated using a HYSPLIT  
 run with release from (43N, 75W)  
 (starting at 83090100, Q=3000g/h)

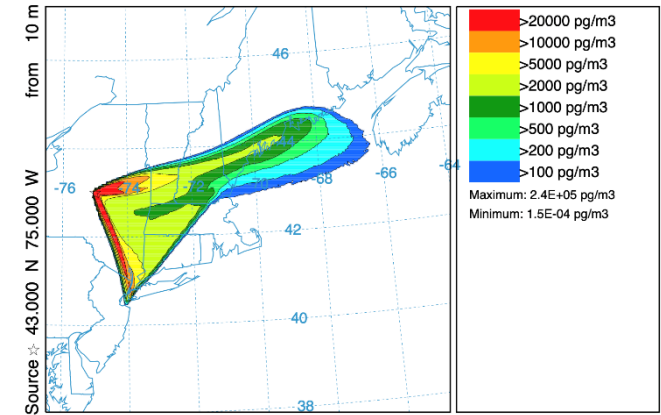
Mean dilution factors are weighted by  
 Measurements (the measured value  
 in the **numerator** of the source rate)

29 inverse runs from each measurement.

Inverse: the source term as 1/M for  
 the calculation

$$Q = M / D \quad \longrightarrow \quad 1/Q = D / M$$

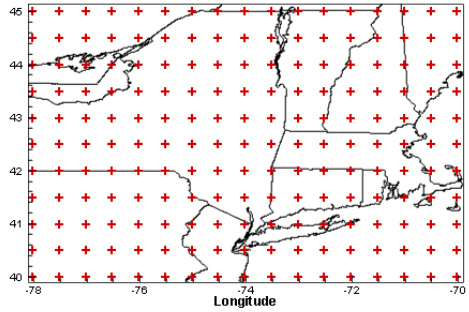
At (43N,75W),  $D/M = 1/Q = 0.584 \times 10^{-15}$   
 $Q = 1.7 \times 10^{15}$  pg/h or about **1700 g/h**



CAPTEX Hypothetical

year	mn	dy	shr	dur	lat	lon	pmch	stn	
1983	9	1	1800	0300	40.8500	-73.9700	445.4	903	
1983	9	1	2100	0300	40.8500	-73.9700	322097.5	903	
1983	9	2	0000	0300	40.8500	-73.9700	114424.1	903	
1983	9	2	0000	0300	41.2700	-73.8000	149894.9	904	
1983	9	2	0000	0300	42.2500	-73.8000	47820.9	906	
1983	9	2	0300	0300	40.8500	-73.9700	181.7	903	
1983	9	2	0300	0300	41.2700	-73.8000	3990.4	904	
1983	9	2	0300	0300	42.2500	-73.8000	3878.4	906	
1983	9	2	0600	0300	43.0500	-74.2000	216939.7	908	
1983	9	2	0900	0300	43.0500	-74.2000	38505.6	908	
1983	9	2	0900	0300	41.9500	-72.3000	208.4	4	
1983	9	2	0900	0300	43.0300	-72.8000	54686.5	6	
1983	9	2	1200	0300	43.0500	-74.2000	88.3	908	
1983	9	2	1200	0300	41.9500	-72.3000	1399.1	4	
1983	9	2	1200	0300	41.7300	-71.4300	7.2	102	
1983	9	2	1500	0300	43.0500	-74.2000	6781.0	908	
1983	9	2	1500	0300	43.0300	-72.8000	11.0	6	
1983	9	2	1500	0300	41.7300	-71.4300	72.2	102	
1983	9	2	1500	0300	42.7000	-71.1700	326.4	104	
1983	9	2	1800	0300	43.0500	-74.2000	6310.8	908	
19839	2	1800	0300		43.0300	-72.8000	762.5	6	
1984	1983	9	2	2100	0300	43.0500	-74.2000	985.0	908
1985	1983	9	2	2100	0300	43.0300	-72.8000	845.8	6
1986	1983	9	3	0000	0300	43.0300	-72.8000	22.1	6
1987	1983	9	3	0300	0300	43.0300	-72.8000	221.3	6
1988	1983	9	3	0600	0300	43.0300	-72.8000	44.2	6
1989	1983	9	3	0600	0300	42.7000	-71.1700	126.5	104
1990	1983	9	3	0900	0300	43.0500	-74.2000	126425.6	908
1991	1983	9	3	0900	0300	42.7000	-71.1700	500.7	104

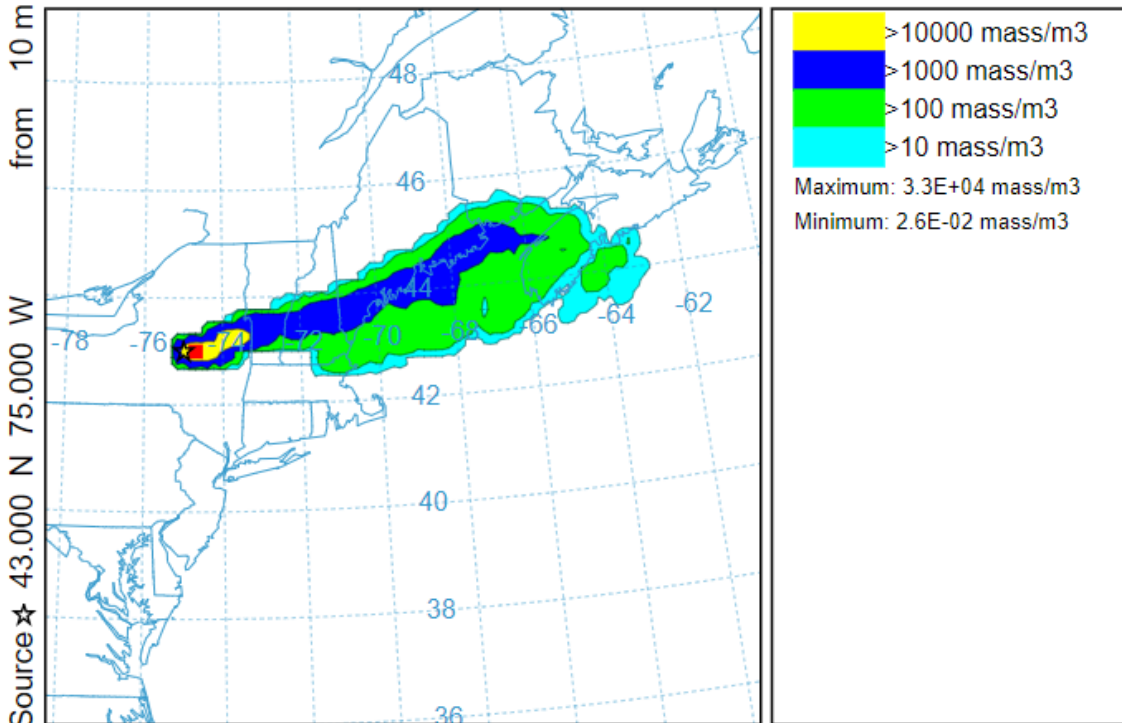
# 13.4 Source-receptor matrix approach



...  
 17x11=187  
 HYSPLIT runs

	A	B	C	D	E	F	G
1		Receptor 1	Receptor 2	Receptor 3	...	Receptor M	
2	Source 1						Rm from S1
3	Source 2						Rm from S2
4	Source 3						Rm from S3
5	...						
6	Source N						Air Concentration Maps
7		Sn to R1	Sn to R2	Sn to R3			
8		Source Sensitivity Maps					

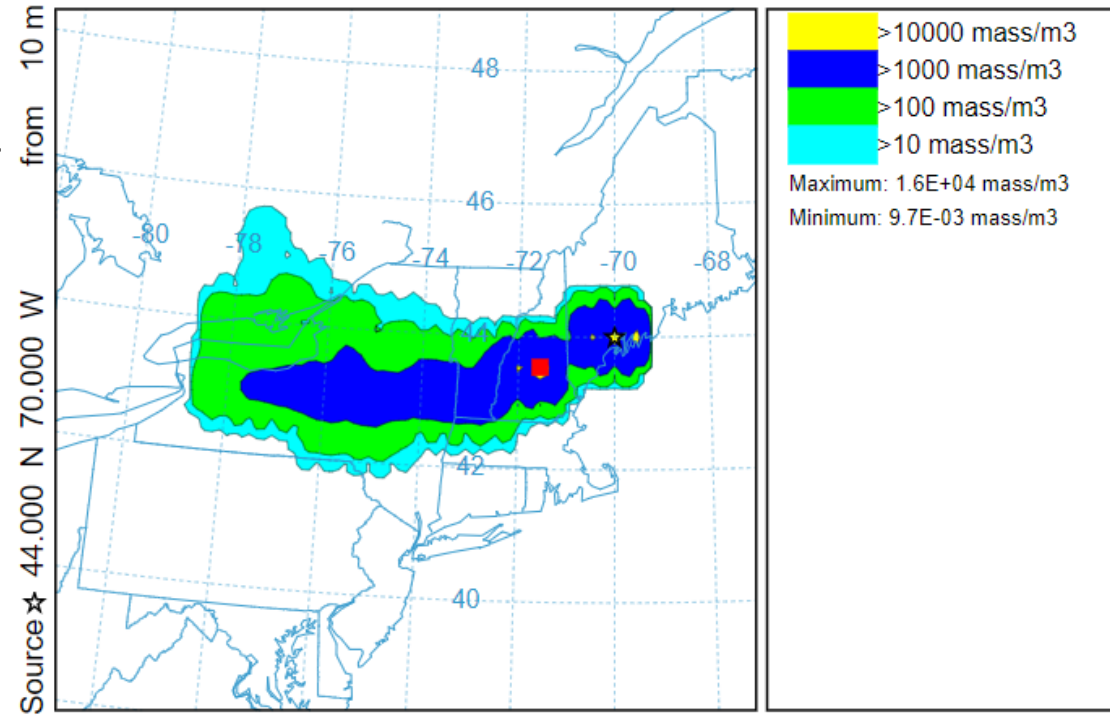
Contributions from the selected Source  
 Air Concentration (mass/m<sup>3</sup>) averaged between 0 m and 100 m  
 Integrated from 0900 03 Sep to 1200 03 Sep 83 (UTC)  
 C(R) Release started at 0000 01 Sep 83 (UTC)



CDC1 METEOROLOGICAL DATA

Dilution factor  
 from  
 (43N, 75W)  
 to  
 (44N 70W)  
 approximately  
 $(1000 \times 10^{-15})$   
 $=10^{-12}$

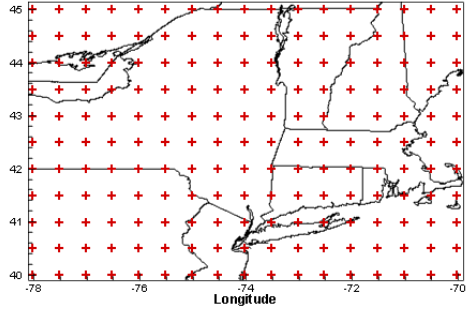
Sources Contributing to the selected Receptor  
 Values (mass/m<sup>3</sup>) averaged between 0 m and 100 m  
 Integrated from 0900 03 Sep to 1200 03 Sep 83 (UTC)  
 C(S) Release started at 0000 01 Sep 83 (UTC)



CDC1 METEOROLOGICAL DATA

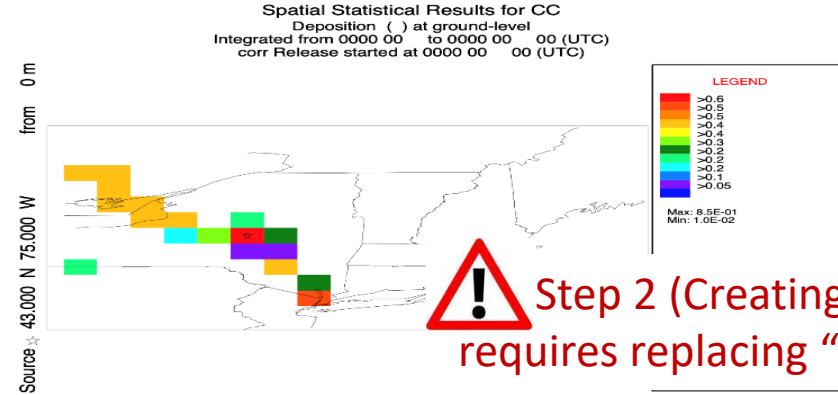


# 13.5 Source Location Statistics



...  
17x11=187  
HYSPLIT runs

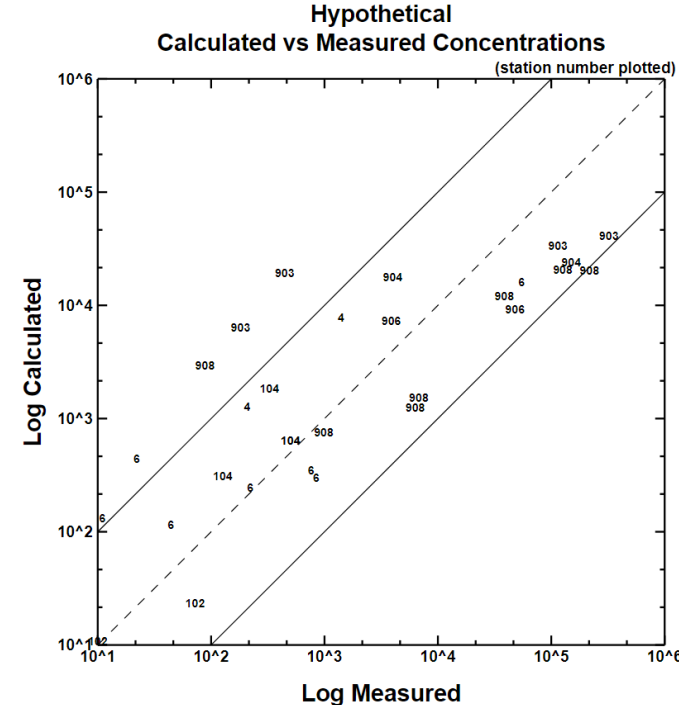
	A	B	C	D	E	F	G
1		Receptor 1	Receptor 2	Receptor 3	...	Receptor M	
2	Source 1	-----					Rm from S1
3	Source 2	-----					Rm from S2
4	Source 3	-----					Rm from S3
5	...						
6	Source N						Air
7		Sn to R1	Sn to R2	Sn to R3			Concentration
8		Source Sensitivity Maps					Maps



(43N, 75W)

37862.18 Average measured concentration  
7836.88 Average calculated concentration  
0.21 Ratio of calculated/measured

Lat	Lon	Corr	NMSE	FB	FMS	KSP	Rank
...							
41.50	-75.00	0.00	999.99	-2.00	0.00	100.00	0.00
42.00	-75.00	-0.15	596.18	-1.97	10.34	87.00	0.27
42.50	-75.00	0.05	40.20	-1.57	51.72	45.00	1.28
43.00	-75.00	0.85	17.01	-1.29	82.76	14.00	2.76
43.50	-75.00	0.20	96.09	-1.81	31.03	66.00	0.78
44.00	-75.00	-0.01	611.09	-1.97	27.59	69.00	0.60
44.50	-75.00	0.02	9678.20	-2.00	3.45	100.00	0.04

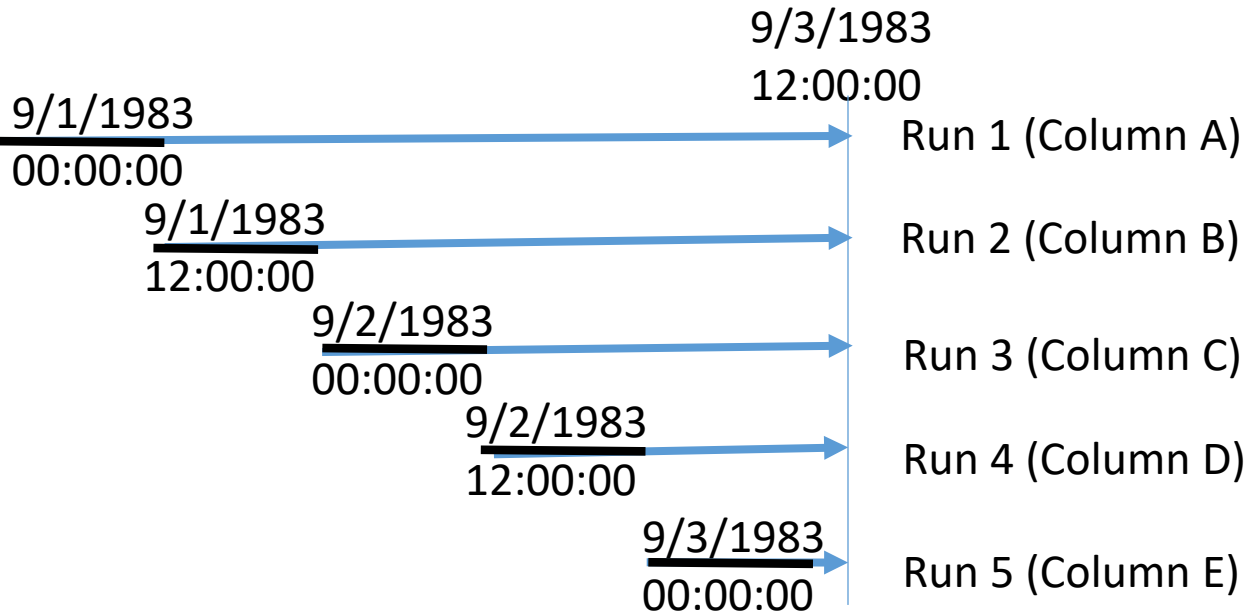


$$Q = 10^{15} \text{ pg/hr} / 0.21 \sim 5000 \text{ g/hr}$$

A significant number of sampling locations will move toward over-prediction if all the points are shifted upward by a factor-of-five.

# 13.6 Solving the coefficient matrix (CM)

Measurements  
(pg/m<sup>3</sup>),  $R_j$



Transfer Coefficient Matrix:  $(D_{ij})^T$

	A	B	C	D	E	F
1	9/1/83 0:00	9/1/83 12:00	9/2/83 0:00	9/2/83 12:00	9/3/83 0:00	9/3/83 12:00
2	1.18E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.45E+02
3	9.75E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.22E+05
4	1.50E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.14E+05
5	4.32E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.50E+05
6	0.00E+00	1.36E+01	0.00E+00	0.00E+00	0.00E+00	4.78E+04
7	1.42E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.82E+02
8	2.21E+00	1.71E-02	0.00E+00	0.00E+00	0.00E+00	3.99E+03
9	0.00E+00	2.76E+00	0.00E+00	0.00E+00	0.00E+00	3.88E+03
10	0.00E+00	0.00E+00	4.69E+01	0.00E+00	0.00E+00	2.17E+05
11	0.00E+00	0.00E+00	4.41E-02	0.00E+00	0.00E+00	3.85E+04
12	0.00E+00	6.51E-02	0.00E+00	0.00E+00	0.00E+00	2.08E+02
13	0.00E+00	1.78E+01	0.00E+00	0.00E+00	0.00E+00	5.47E+04
14	0.00E+00	0.00E+00	4.39E-03	0.00E+00	0.00E+00	8.83E+01
15	5.37E+00	1.11E+01	0.00E+00	0.00E+00	0.00E+00	1.40E+03
16	0.00E+00	4.75E-02	0.00E+00	0.00E+00	0.00E+00	7.20E+00
17	0.00E+00	0.00E+00	3.07E-02	1.01E+00	0.00E+00	6.78E+03
18	0.00E+00	1.34E-01	0.00E+00	0.00E+00	0.00E+00	7.22E+01
19	0.00E+00	5.62E-01	0.00E+00	0.00E+00	0.00E+00	3.26E+02
20	0.00E+00	0.00E+00	0.00E+00	4.39E-01	0.00E+00	6.31E+03
21	0.00E+00	0.00E+00	4.38E-03	1.19E-01	0.00E+00	7.62E+02
22	0.00E+00	0.00E+00	0.00E+00	4.39E-03	0.00E+00	9.85E+02
23	0.00E+00	0.00E+00	0.00E+00	1.58E-01	0.00E+00	8.46E+02
24	0.00E+00	0.00E+00	0.00E+00	2.19E-02	0.00E+00	2.21E+02
25	0.00E+00	0.00E+00	0.00E+00	2.63E-02	0.00E+00	4.42E+01
26	0.00E+00	0.00E+00	0.00E+00	1.18E-01	0.00E+00	1.26E+02
27	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.33E+01	1.26E+05
28	0.00E+00	0.00E+00	0.00E+00	2.40E-01	0.00E+00	5.01E+02
29						

$$\sum_{i=1}^5 (D_{ij} S_i) = R_j$$

Get  $S_i$  by using  
Singular value decomposition (SVD)

```
source.txt file contents:
    Date,      Result,
30560.000,    3.353E+03,
30560.500,    2.291E+03,
30561.000,    4.628E+03,
30561.500,    7.430E+03,
30562.000,    9.474E+03,
```

# 13.7 Cost Function Minimization of the CM

Transfer Coefficient Matrix:  $(D_{ij})^T$



Measurements

$(\text{pg}/\text{m}^3), \downarrow C_j$



	A	B	C	D	E	F
1	9/1/83 0:00	9/1/83 12:00	9/2/83 0:00	9/2/83 12:00	9/3/83 0:00	9/3/83 12:00
2	1.18E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.45E+02
3	9.75E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.22E+05
4	1.50E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.14E+05
5	4.32E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.50E+05
6	0.00E+00	1.36E+01	0.00E+00	0.00E+00	0.00E+00	4.78E+04
7	1.42E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.82E+02
8	2.21E+00	1.71E-02	0.00E+00	0.00E+00	0.00E+00	3.99E+03
9	0.00E+00	2.76E+00	0.00E+00	0.00E+00	0.00E+00	3.88E+03
10	0.00E+00	0.00E+00	4.69E+01	0.00E+00	0.00E+00	2.17E+05
11	0.00E+00	0.00E+00	4.41E-02	0.00E+00	0.00E+00	3.85E+04
12	0.00E+00	6.51E-02	0.00E+00	0.00E+00	0.00E+00	2.08E+02
13	0.00E+00	1.78E+01	0.00E+00	0.00E+00	0.00E+00	5.47E+04
14	0.00E+00	0.00E+00	4.39E-03	0.00E+00	0.00E+00	8.83E+01
15	5.37E+00	1.11E+01	0.00E+00	0.00E+00	0.00E+00	1.40E+03
16	0.00E+00	4.75E-02	0.00E+00	0.00E+00	0.00E+00	7.20E+00
17	0.00E+00	0.00E+00	3.07E-02	1.01E+00	0.00E+00	6.78E+03
18	0.00E+00	1.34E-01	0.00E+00	0.00E+00	0.00E+00	7.22E+01
19	0.00E+00	5.62E-01	0.00E+00	0.00E+00	0.00E+00	3.26E+02
20	0.00E+00	0.00E+00	0.00E+00	4.39E-01	0.00E+00	6.31E+03
21	0.00E+00	0.00E+00	4.38E-03	1.19E-01	0.00E+00	7.62E+02
22	0.00E+00	0.00E+00	0.00E+00	4.39E-03	0.00E+00	9.85E+02
23	0.00E+00	0.00E+00	0.00E+00	1.58E-01	0.00E+00	8.46E+02
24	0.00E+00	0.00E+00	0.00E+00	2.19E-02	0.00E+00	2.21E+02
25	0.00E+00	0.00E+00	0.00E+00	2.63E-02	0.00E+00	4.42E+01
26	0.00E+00	0.00E+00	0.00E+00	1.18E-01	0.00E+00	1.26E+02
27	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.33E+01	1.26E+05
28	0.00E+00	0.00E+00	0.00E+00	2.40E-01	0.00E+00	5.01E+02
29						

$$F(S) = \sum_{i=1}^5 \frac{(S_i - S_{ib})^2}{\sigma_{ij}^2} + \sum_{j=1}^{27} \frac{(R_j^h - R_j^o)^2}{\varepsilon_{ij}^2}$$

$$R_j^h = \sum_{i=1}^5 (D_{ij} S_i)$$



Get  $S_j$  by minimizing  $F$

source.txt file contents:

```
30560.0000 1.3808E+03
30560.5000 1.3165E+03
30561.0000 3.0695E+03
30561.5000 3.5270E+03
30562.0000 2.9136E+03
```



# Source attribution methods – Summary

$$C(x, y, z, t) = \mathbf{D}(x, y, z, t) \times S(x, y, z, t)$$

- A single model run to estimate emission rate (a crude estimation)
  - 13.1 forward dispersion
  - 13.2 forward/backward dispersion
- A series of backward dispersion from measurement location/time (13.3)
  - Measurements as numerators to estimate release location
  - Measurements as denominator for (1/Q or 1/S)
- Matrix runs – 187 forward runs from different locations (13.4, 13.5)
  - Using statistics to infer likely release location
  - Estimate emission rate from the inferred release location
- Find emission temporal variations using a Coefficient Matrix (CM or TCM)
  - Using SVD to solve source terms
  - Minimizing a cost function to estimate the emissions (considering model and observation uncertainties)